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USING AN EXTENDED IMPEDANCE MEASUREMENT METHOD FOR THE ESTIMATION OF POROSITY AND FLOW RESISTANCE OF POROUS MATERIALS.

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Abstract

Today in the most cases the acoustic properties of porous material are characterized by the absorption coefficient or the surface impedance. Therefore measurement are carried out by use of Kundt's- or the Impedance Tube method.

An other approach to characterize the acoustical properties could be the determination of a pair of characteristic values, e.g. the complex characteristic impedance and the complex wave number. By use of this pair the description of the sound field inside of the absorber is feasible. So for instance the transmission loss of an absorbing layered duct or of an absorbing layer as well as the absorption coefficient of porous material in dependence of the thickness of the layer can be predicted.

Furthermore the measurement of the pair of the complex values allows the estimation of parameters of the porous material, e.g. the porosity or the flow resistance. These parameters have been estimated by least square fitting using absorber models as the Delany/Bazely Model, the model of the Homogeneous Media or the Phenomenological Model. The results of the estimation have been compared with the results of direct measurements. Here especially for the one parameter model (Delany/Bazely Model) a good agreement has been observed.

INTRODUCTION

The acoustical behavior of homogeneous and isotropic porous media can be described by the use of a pair of characteristic values, e.g. the complex char-

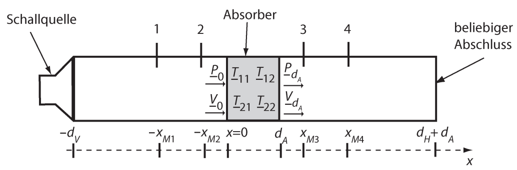


Figure 1: Principle of the Transfer-Matrix-Method.

acteristic impedance $\underline{Z}_A$ and the complex wave number $\underline{k}_A$. So the prediction of the absorption coefficient of a layer of porous sound absorbing material for different angles of incidence is feasible. Moreover the prediction can be carried out in dependence of the thickness of this layer or for diffuse sound incidence, as measured in a reverberation room.

There are several acoustical models of porous sound absorbing material, which allow the calculation of the characteristic values as function of the frequency. Here as input values the absorber parameters, e.g. porosity, tortuosity or flow resistance, are used.

In some cases the direct measurement of the absorber parameters is very extensive. So for the prediction of the acoustical behavior of porous asphalt the tortuosity has been measured by use of an hydro-electrical analogy. Here the sample of porous asphalt has to be submerge in water. This procedure is very time consuming and inaccurate. Therefore the estimation of absorber parameters using measured characteristic values was desirable.

MEASUREMENT OF THE CHARACTERISTIC VALUES

For the determination of the pair of the characteristic values the Transfer-Matrix-Method (TMM) of SON and BOLTON [1] has been employed. This technique bases on the use of four microphones positioned in a tube, two in front and two behind of the sample under examination (see Fig. 1). Inside of the tube the propagation of plane waves is assumed. Thus the tube can be considered as one-dimensional wave guide consisting of different sections. Therefore the coefficients T_{xy} of the transfer matrix of the absorber can be calculated by the sound pressure $\underline{p}$ and sound velocity $\underline{\mathrm{v}}$ in front and behind of the sample, at x=0 and x=d respectively:

$$\begin{bmatrix} \underline{p} \\ \underline{\mathrm{v}} \end{bmatrix}_{x=0} = \begin{bmatrix} \underline{T}_{11} & \underline{T}_{12} \\ \underline{T}_{21} & \underline{T}_{22} \end{bmatrix} \begin{bmatrix} \underline{p} \\ \underline{\mathrm{v}} \end{bmatrix}_{x=d} . \tag{1}$$

The wave propagation in the absorber sample has to be described by the characteristic values $\underline{Z}_A$ and $\underline{k}_A$:

$$\begin{bmatrix} \underline{T}_{11} & \underline{T}_{12} \\ \underline{T}_{21} & \underline{T}_{22} \end{bmatrix} = \begin{bmatrix} \cos \underline{k}_A d & j\underline{Z}_A \sin \underline{k}_A d \\ \frac{1}{\underline{Z}_A} j \sin \underline{k}_A d & \cos \underline{k}_A d \end{bmatrix} . \tag{2}$$

So the complex characteristic wave number and the complex characteristic impedance is:

$$\underline{k}_A = \frac{1}{d_A} \arccos(\underline{T}_{11}) \tag{3}$$

and

$$\underline{Z}_A = \left(\frac{\underline{T}_{12}}{\underline{T}_{21}} \right)^{\frac{1}{2}} \tag{4}$$

respectively.

Furthermore the wave propagation in front and behind the sample is predicted using the characteristic values of air, $\underline{k}_0$ and Z_0. Here for the improvement of the accuracy of the TMM-technique the wave number has been considered as complex value. So the damping of the sound waves at higher frequencies was taken into account.

The arccos-function in Eq. (3) is an analytic and many-valued function. This can be shown by

$$\arccos(\underline{T}_{11}) = -j\mathrm{Ln}\left(\underline{T}_{11} + j\sqrt{1 - \underline{T}_{11}^2} \right) \tag{5}$$

and with $\underline{\rho} = \underline{T}_{11} + j\sqrt{1 - \underline{T}_{11}^2}$

$$\mathrm{Ln}(\underline{\rho}) = \ln \left| \underline{\rho} \right| + j \left(\arg(\underline{\rho}) + 2k\pi \right) \text{ with } k \in Z. \tag{6}$$

The solution of Eq. (6) lies in the RIEMANN-surface S. S is a surface-like configuration that covers the complex plane with k branches. The branch cuts of Eq. (3) are at $\mathbf{Im}\{\underline{T}_{11}\} = 0$. Moreover in accordance with BREKHOVSKIKH [2] for an inhomogeneous plane wave with

$$\underline{p} = \underline{A}_0 e^{-j\underline{k}_A} \tag{7}$$

the imaginary part of $\underline{k}_A$ is $\mathbf{Im}\{\underline{k}_A\} > 0$. So if

$$\mathbf{Im}\{\underline{T}_{11}\} < 0 \tag{8}$$

then the solution of the imaginary part of Eq. (6) is

$$\mathbf{Im}\{\underline{k}_A\} = -\frac{1}{d_A} \mathbf{Im}\{\arccos \underline{T}_{11}\}. \tag{9}$$

Furthermore with (8) and a phase shift of π the real part of $\underline{k}_A$ is

$$\mathbf{Re}\{\underline{k}_A\} = \frac{1}{d_A} (2\pi - \mathbf{Re}\{\arccos \underline{T}_{11}\}). \tag{10}$$

By using the technique on several absorbing materials measurements have been carried out. An example is shown in Fig. 2. Here metallic hollow sphere structure have been examined. In the lower graph at the left hand side of Fig. 2 an

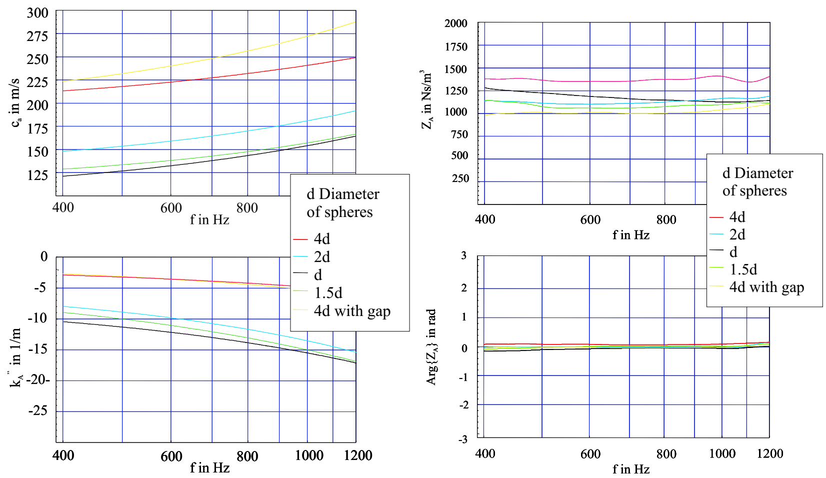


Figure 2: Measurement of $\underline{k}_A$ and $\underline{Z}_A$ of different metallic hollow sphere structures using the Transfer-Matrix-Method. Here $\underline{c}_a$ is the sound speed of an inhomogeneous plane wave inside of the structures, $\mathbf{Re}\{\underline{k}_A\} = \omega/\underline{c}_a$. The value $\mathbf{Im}\{\underline{k}_A\}$ ($\underline{k}_A^{''}$) is equivalent to the damping of a propagating inhomogeneous plane wave inside of the structure.

increase of damping with a decrease of the diameter of the spheres can be observed. The upper graph shows the sound speed of the propagating wave for different sphere diameter. Here an increase of diameter leads to an increase of the sound speed. The left hand graphs show the measured results for the characteristic impedance. According to the upper graph on the left hand side a decrease of the diameter of the spheres causes a decrease of the magnitude of the characteristic impedance. Furthermore the phase of the characteristic impedance is nearly independent on the sphere diameter.

The measured characteristic values ($\underline{k}_A$ and $\underline{Z}_A$) have been used for the calculation of the absorption coefficient for diffuse sound incidence. In Fig. 3 the absorption coefficient measured in the reverberation room according to ISO 354 is shown in comparison to the calculated values obtained by the characteristic values. The diffraction on the edges has been considered [4] in the calculation. The graphs of both values show a good agreement at higher frequencies (f>800 Hz).

ESTIMATION

The estimation of the parameters of an absorber, e.g. the porosity σ or the air flow resistivity Ξ, can be carried out using the measured $\underline{y}_f^{meas}$ and the predicted $\underline{y}_f^{pred}$ characteristic values. For that purpose different methods of optimization, e.g. the "maximum-likelihood-method" or the "least square method", have been examined. Thereby the best results could be observed by the use of the "least square method". The deviation between the measured characteristic values and the predicted values shall be $\varepsilon_f = \underline{y}_f^{meas} - \underline{y}_f^{pred}$. So the error function defined by

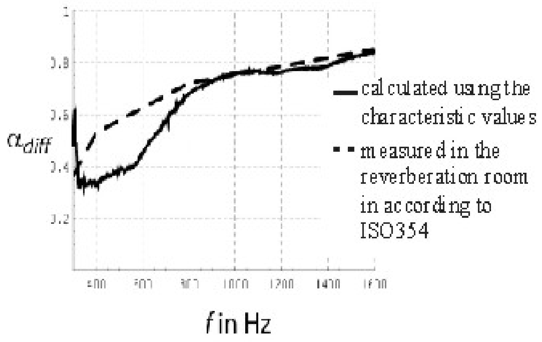


Figure 3: The absorption coefficient for diffuse sound incidence calculated using the characteristic values ($\underline{k}_A$, $\underline{Z}_A$) compared to the measured values (reverberation room in according to ISO 354) .

the "least square method" is

$$J(\theta) = \sum_{f=1}^{n} \underline{\varepsilon}_f(\theta)\underline{\varepsilon}_f^*(\theta). \tag{11}$$

Here the parameter f stands for the frequency. The criteria of the optimization is

$$min_{x\in\mathbf{R}} J(\theta). \tag{12}$$

For the examinations the following models have been used.

- The model of Delany/Bazeley [3]:

$$\frac{\underline{k}_A}{\underline{k}_0} = 1 + a''C^{\alpha''} - ja'C^{\alpha'}, \tag{13}$$

$$\frac{\underline{Z}_A}{\underline{Z}_0} = 1 + b'C^{\beta'} - jb''C^{\beta''}, \tag{14}$$

  with $C = \Xi/(\rho_0 f) = 1/E$ and the parameters e.g. mineral for wool:

$$\begin{array}{ll} a' = 0,189, & \alpha' = 0,595, \\ a'' = 0,0978, & \alpha'' = 0,700, \\ b' = 0,0571, & \beta' = 0,754, \\ b'' = 0,087, & \beta'' = 0,732. \end{array}$$

- The model of the "Homogeneous Media"[3]:

$$\frac{\underline{k}_A}{\underline{k}_0} = \sqrt{\frac{\kappa + jE/E_0}{1 + jE/E_0}\left(\tau - j\frac{\sigma}{2\pi E}\right)} \tag{15}$$

$$\frac{\underline{Z}_A}{\underline{Z}_0} = \frac{1}{\sigma}\sqrt{\frac{1 + jE/E_0}{\kappa + jE/E_0}\left(\tau - j\frac{\sigma}{2\pi E}\right)} \tag{16}$$

with

$$E = \frac{\rho_0 f}{\Xi} \qquad E_0 = \frac{\rho_0 f_0}{\Xi} \qquad 2\pi f_0 \Upsilon = 1 \qquad \kappa = 1.4 \tag{17}$$

- and the "Phenomenological Model" [5]:

$$\frac{\underline{k}_A}{k_0} = \sqrt{\tau\kappa}\sqrt{1 - j\frac{f_\mu}{f}}\sqrt{1 - \left(1 - \frac{1}{\kappa}\right)\frac{1}{1 - j\frac{f_\theta}{f}}} \tag{18}$$

$$\frac{\underline{Z}_A}{Z_0} = \frac{1}{\sigma}\sqrt{\frac{\tau}{\kappa}}\frac{\sqrt{1 - j\frac{f_\mu}{f}}}{\sqrt{1 - \left(1 - \frac{1}{\kappa}\right)\frac{1}{1 - j\frac{f_\theta}{f}}}} \tag{19}$$

$$f_\mu = \frac{\Xi\sigma}{2\pi\rho_0\tau} \qquad f_\theta = \frac{\Xi}{2\pi\rho_0 \mathrm{Pr}} \tag{20}$$

In the Eq. (13)-(20) the parameter Pr is the "Prandtl-Number", ρ_0 stands for the atmospheric pressure of air, τ is the tortuosity and Υ is the time of relaxation.

The calculation of the function $J(\theta)$ requires the prediction of the characteristic values ($\underline{k}_A$ and $\underline{Z}_A$) on the basis of the aforementioned models. So the functions $\underline{y}_f^{pred}$ in Eq. (11) are nonlinear. The solution of (12) can be found by nonlinear programming. For it several methods have been examined, e.g. the "Method of Steepest Descent", the "Newton-Raphson-Method" and the "Levenberg-Marquardt-Method". Here it turns out, that the best performance is provided by the "Levenberg-Marquardt-Method" (e.g. no instabilities due to singularities of the "Hesse-Matrix" because of the estimation of that matrix). The "Levenberg-Marquardt-Method" uses an iterative process to approach one root of a function. The specific root that the process locates depends on the initial chosen value (local convergence). Therefore to reach reliable results a good initial guess is necessary.

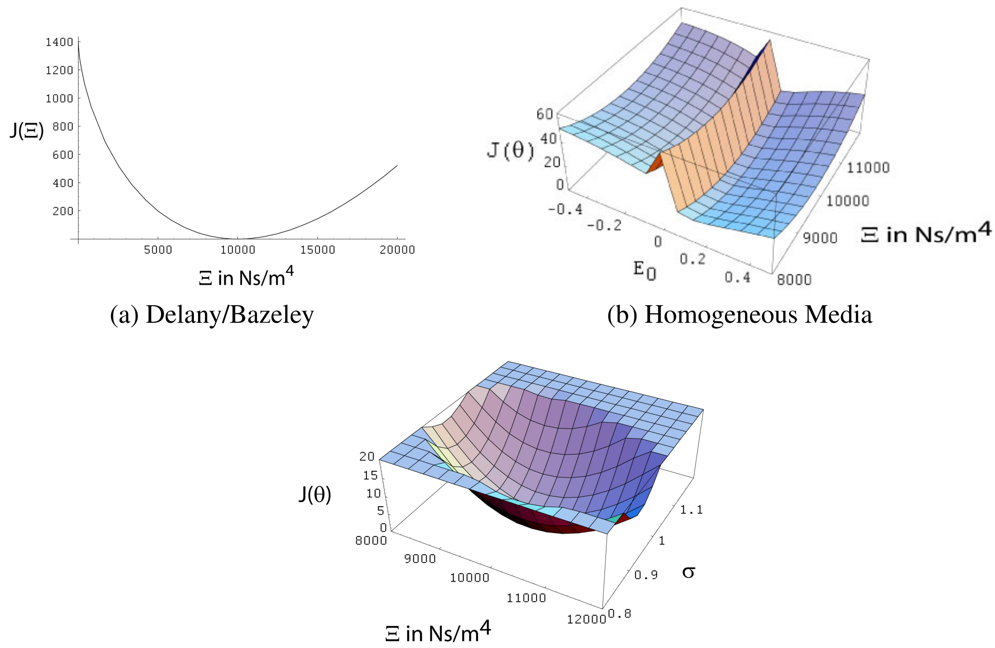
In Fig. (4) the graphs of the function $J(\theta)$ are plotted for different models of the absorber. It can be seen, that for the model of Delany/Bazeley (a) and the "Phenomenological Model" (c) there is a global minimum only. For this case the "Levenberg-Marquardt-Method" will converge. In the plot (b) of Fig. (4) a singularity at E_0 occurs . Here saddle points of the function can be observed. Thus the calculation of an initial guess is necessary.

This can be carried out by the use of the equations of the model of "Homogeneous Media", Eq. (15) and (16) [3]. By means of

$$\underline{k}_A\underline{Z}_A/(Z_0 k_0) = \frac{1}{\sigma}\left(\chi - j\frac{\sigma}{2\pi E}\right), \tag{21}$$

with the Eq. (17) of the absorber number E and

$$\mathrm{Im}\{\underline{k}_A\underline{Z}_A/(Z_0 k_0)\} = \frac{1}{2\pi E} \tag{22}$$



(a) Delany/Bazeley

(b) Homogeneous Media

(c) Phenomenological Model

Figure 4: The Function $J(\theta)$ defined in Eq. (11) shown for the considered models of the absorber.

the initial guess for the air flow resistivity can be calculated by

$$\Xi = 2\pi\rho_0 f \,\mathrm{Im}\left\{\underline{k}_A \underline{Z}_A/(Z_0 k_0)\right\}. \tag{23}$$

Furthermore the quotient of $\underline{k}_A$ and $\underline{Z}_A$

$$\frac{\underline{k}_A Z_0}{\underline{Z}_A k_0} = \sigma\left(\frac{\kappa + E^2/{E_0}^2}{1+E^2/{E_0}^2} + j\frac{E/E_0(1-\kappa)}{1+E^2/{E_0}^2}\right) \tag{24}$$

and

$$\lim_{f\to\infty}\left(\frac{\kappa + E^2/{E_0}^2}{1+E^2/{E_0}^2}\right) = 1 \tag{25}$$

allows the prediction of the initial guess of the porosity

$$\sigma \approx \mathrm{Re}\left\{\frac{\underline{k}_A/k_0}{\underline{Z}_A/Z_0}\right\}\Bigg|_{f\to\infty}. \tag{26}$$

As aforementioned the function $J(\theta)$ calculated for the "Phenomenological Model" has a global minimum. So the iterative process should converge after a finite number of steps. But for moderate number of the iterative steps initial guesses for the air flow resistivity Ξ, the porosity σ and the tortuosity τ have been derived:

$$\Xi = -\mathrm{Im}\left\{\underline{k}_A \underline{Z}_A/(Z_0 k_0)\right\} 2\pi\rho_0 f, \tag{27}$$

$$\sigma = \frac{\mathrm{Re}\left\{(\underline{k}_A Z_0)/(\underline{Z}_A k_0)\right\}}{\kappa - \frac{\kappa-1}{1+\left(\frac{\Xi}{2\pi\rho_0 \mathrm{Pr} f}\right)^2}} \tag{28}$$

and

$$\tau = \sigma \mathrm{Re}\left\{\underline{k}_A \underline{Z}_A/(Z_0 k_0)\right\}. \tag{29}$$

CONCLUSION

In Tab. 1 the relative error of the estimated values of the air flow resistivity is plotted. The relative errors have been calculated applying directly measured values of flow resistivity. Here for the examined structures good results can be observed by the estimation based on the model of DELANY/BAZELY. The ”Phenomenological Model” was developed for the prediction of the acoustical behavior of porous roads. This could explain the bad results ($\Delta\Xi/\Xi \approx 10-13\%$) of the estimation using this model. One reason of the error observed by the estimation using the model of the ”Homogeneous Media” ($\Delta\Xi/\Xi \approx 10-13\%$) could be the rough assumption for the prediction of the air flow resistivity.

	$\Delta\Xi/\Xi$ mineral wool	$\Delta\Xi/\Xi$ open porous foam
DELANY/BAZELY	5.1%	5,9%
Homogeneous Media	13.2%	10.8
Phenomenological Model	13.0%	10.6%

Table 1: The relative error $\Delta\Xi/\Xi$ of the estimated value of air flow resistivity calculated on the base of different absorber models. The relative errors have been calculated by direct measured values of flow resistivity. These measurements have been carried out in accordance with ISO 9053 (mineral wool Ξ=6100 Ns/m^4, open porous foam Ξ=6070 Ns/m^4).

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