

Modeling of Porous Asphalt as Extended Reacting Absorber using the Transmission-Line-Method (TLM)

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It is assumed that the primary sound sources occurring during the tyre/road interaction are located very close to the road surface. The “Weyl von de Pol” (W. v. P.) solution enables the description of that kind of sound field above an impedance plane (porous road surface). One of the basic requirements of the aforementioned theory is a local reacting absorbing layer. But in the case of an extended reacting layer the combination of the W.v.P-solution with the so called “effective impedance” concept seems to be feasible. Furthermore by different authors it was pointed out that this concept can fail in the case of small values of the flow resistivity.

The determined characteristic values of the porous road surface, the complex characteristic impedance and the complex wave number, lead to the assumption that the road surface has to be modeled as extended reacting. For the examination of the extended reaction a TLM-model has been set up. This model was extended to lossy media. In difference to other TLM-models of lossy media here the consideration of both characteristic values has been enabled. The results have been compared with that of an analytical model. In spite of deviations between the predictions a model with a spherical wave front at nearly grazing sound incidence has been set up. Here the extended reaction of the absorber can be observed. Moreover an attenuation of -20 dB at the discrete frequency of 1000 Hz has been predicted. This attenuation occurs, when the wave length within the absorber is approximately a quarter of the thickness of the hard backed absorber layer. In future the accuracy of the model has to be examined systematically.

1. INTRODUCTION

For the optimization of porous road surface an analytical model has been established. Here the porous pavement is understood as a homogeneous porous material in the frequency range of interest. The dominating sound sources of tyre/road noise are located very close to the asphalt surface. That means spherical wave fronts can not be assumed as plane waves. For the description of spherical wave fronts above an impedance plane the theory of “Weyl - von de Pol” is feasible e.g. [1]. However a “local reacting” absorber is a basic requirement for the validity of the derivation in [1]. One criterium of the existence of a local reacting absorber is the refraction index. In **Figure 1** the limit between local and extended reaction is shown. This prediction has been carried out using the Equation (10) in [2]. The red and blue curve show the limit of data won from a pool of about 250 “Marshall” specimen. So, if an asphalt layer of a thickness $d_m=0.04$ m is used at 1000 Hz the layer has to be assumed as extended reacting.

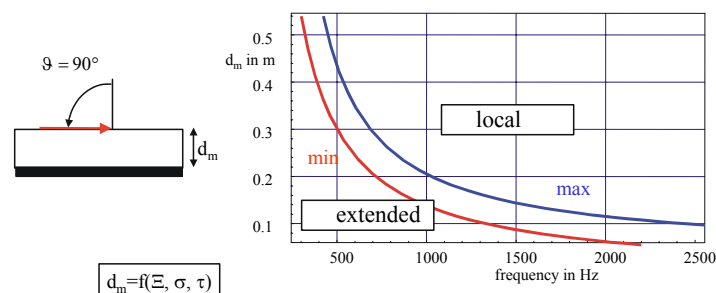


Figure 1. Extended or local reacting absorber calculated for grazing incidence. Prediction using Equation (10) in [2] for a pool of 250 “Marshall” specimen. The blue and the red line are the limits of the calculated data .

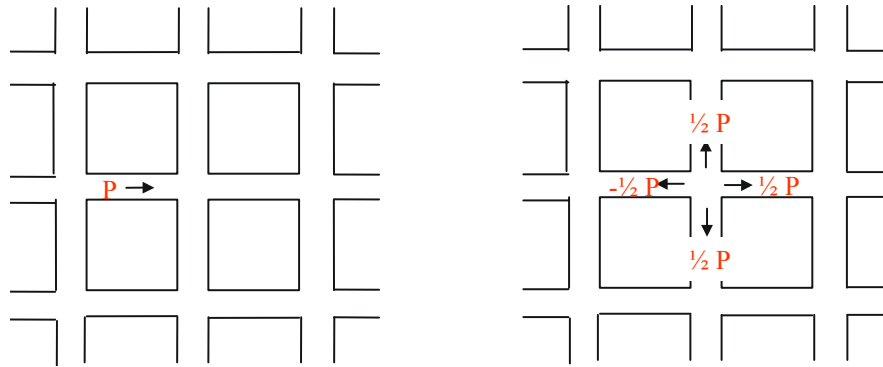


Figure 2.2-D TLM mesh [4].

Generally according to **Figure 1** it is pointed out that in the frequency range of interest (500-2500 Hz) for a layer thickness smaller than $0.1m$ the porous road has to be modeled as extended reacting absorber. The wave number k_l used in Equation (10) of reference [2] has been calculated using the “Phenomenological Model” [3] and is understood as complex quantity.

In [2] a heuristic extension of the classic “Weyl von de Pol” solution is proposed using an effective admittance. In this reference the predicted data were compared with numerical calculations and measured data. It was pointed out that there is less satisfactory agreement between numerical calculation and the heuristic extension when both the source and the receiver are closed to the impedance plane (0.01m), when the distance source-receiver is small (2m) and when the absorber has a small flow resistivity (1000-8000 Ns/m⁴).

Unfortunately in the tyre/road source model the sound sources are very closed to the ground and the measured values of flow resistivities are very small. Therefore the validity of the extension of the classical “Weyl von de Pol” theory has to be examined for the aforementioned constellation. A model was needed which enables the description of wave propagation in a lossless medium (air) and in a lossy medium (porous road surface) and the interaction of both sound fields. Here the Transmission-Line-Matrix- Model (TLM) seems to be feasible.

2. THE 2-D TRANSMISSION-LINE-MATRIX-MODEL

The 2D TLM-Model consists of a mesh of lines interconnected at nodes as shown in **Figure 2**. In this mesh the energy of an incident pulse with the amplitude P (left hand side of **Figure 2**) is conserved by isotropically spreading (right hand side of **Figure 2**) [4]. This spreading is described by a scattering matrix S .

The inductance L and the capacitance C (see **Figure 4**) of the basic transmission line element is determined by the distance of nodes Δl and the calculating time step Δt . Furthermore the scattering matrix S can be derived by the theory of linear networks [4]. Here in according to the “Thevenin Theorem” each line element can be replaced by its Thevenin equivalent circuit. Using this circuit the scattering matrix S can be calculated (see **Figure 4**).

The scattering matrix considers no losses and the sound propagation speed in the whole mesh is constant. However a model of sound propagation above and within an absorbing layer should include both, a lossless and a lossy region. In the lossy region the sound speed (propagation constant γ) and the amount of loss (damping ratio α) is different from these within the lossless region. The consideration of losses in the basic line elements isn’t possible. Therefore the properties of the lossy region have been included at the nodes of the mesh by an additional capacitance, inductance, resistance and conductance (see **Figure 4**). This elements can be considered by so called stubs [4]. Here the capacitance and the inductance were modeled by the Thevenin equivalent circuit of the basic transmission line element terminated by an open - or short circuit, respectively. The secondary values of a transmission line, the characteristic impedance and the propagation constant can be calculated using Equation (1)

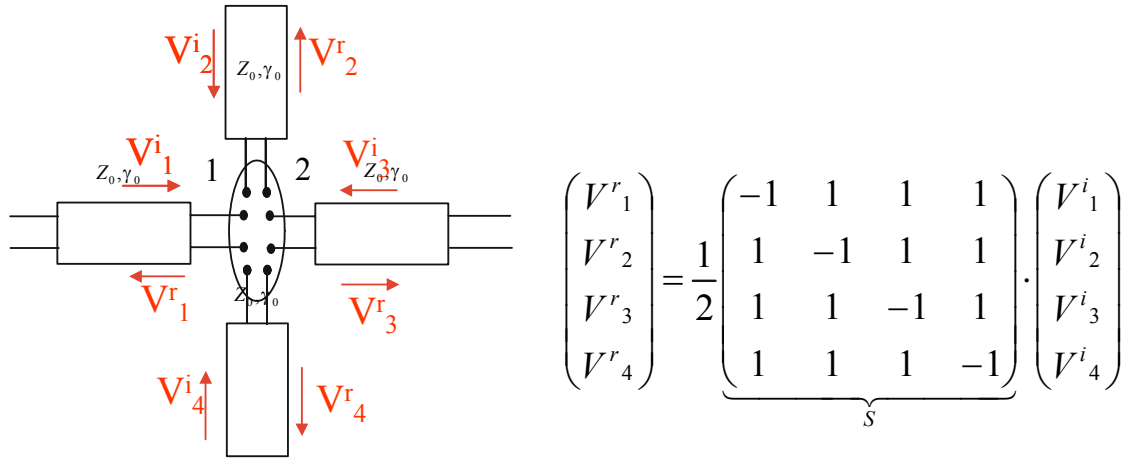


Figure 3. The shunt node and the scattering matrix of a 2-D transmission line mesh.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z| \cdot e^{j\vartheta} \quad (1)$$

The secondary values of the lossy section can be estimated using an absorber model e.g. [3] or measured e.g. [5]. Moreover the separation of the real and the complex part of the values gives the primary quantities of the shunt nodes in the transmission line matrix model:

$$R = |Z| \sqrt{\alpha^2 + \beta^2} \cos\left(\arctan\left(\frac{\beta}{\alpha}\right) + \vartheta\right)$$

$$L = \frac{|Z| \sqrt{\alpha^2 + \beta^2}}{\omega} \sin\left(\arctan\left(\frac{\beta}{\alpha}\right) + \vartheta\right)$$

$$G = \frac{\sqrt{\alpha^2 + \beta^2}}{|Z|} \cos\left(\arctan\left(\frac{\beta}{\alpha}\right) - \vartheta\right)$$

$$C = \frac{\sqrt{\alpha^2 + \beta^2}}{|Z|\omega} \sin\left(\arctan\left(\frac{\beta}{\alpha}\right) - \vartheta\right) \quad (2)$$

By the introduction of additional stubs for the capacitance C and the inductance L and consideration of the elements resistance R and conductance G the scattering matrix yields to Equation (3). As mentioned the stub model for the capacitor has been built by use of an transmission line terminated by an open circuit. Now the Thevenin equivalent circuit consists of an equivalent source and the characteristic admittance Y_s . This is the same for the inductance. Here the transmission line terminated by a short circuit is used. The equivalent Thevenin circuit consist of the equivalent source and a characteristic impedance Z_L .

At this stage of examination the scattering matrix can be used for the computing at discrete frequencies only.

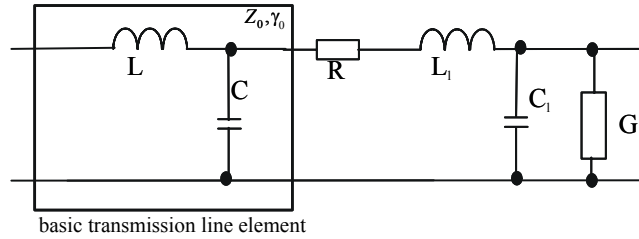


Figure 4. The transmission line element with additional elements for the consideration of losses. The basic element is a lossless transmission line.

$$S = \frac{1}{4 - (G - Y)(R + Z_L + 1)} \begin{pmatrix} XX & 2 & 2 & 2 & -2 & -2 & -2 & -2 & X \\ 2 & XX & 2 & 2 & -2 & -2 & -2 & -2 & X \\ 2 & 2 & XX & 2 & -2 & -2 & -2 & -2 & X \\ 2 & 2 & 2 & XX & -2 & -2 & -2 & -2 & X \\ 2 & 2 & 2 & 2 & XXX & -2 & -2 & -2 & X \\ 2 & 2 & 2 & 2 & -2 & XXX & -2 & -2 & X \\ 2 & 2 & 2 & 2 & -2 & -2 & XXX & -2 & X \\ 2 & 2 & 2 & 2 & -2 & -2 & -2 & XXX & X \\ 2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & XXXX \end{pmatrix}$$

$$X = 2Y(R + Z_L + 1)$$

$$XX = -2 + GY(R + Z_L + 1)$$

$$XXX = -6 + (G - Y)(R + Z_L + 1)$$

$$XXXX = -4 + GY(R + Z_L + 1)$$

(3)

3. EXAMPLE

For a first validation the perpendicular incidence of an plane sound wave on an absorbing layer was calculated and compared with the analytical solution Equation (4):

$$Z_w = Z_1 \cdot \tanh(k_1 d). \quad (4)$$

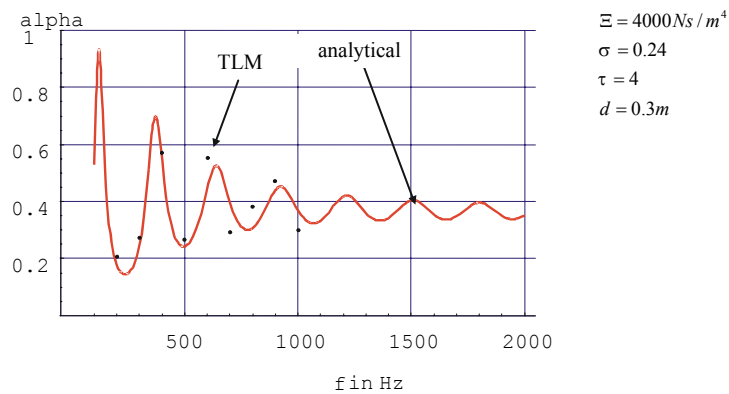


Figure 5. Perpendicular sound incidence of an absorbing layer with thickness 0.3 calculated using the analytical description and TLM.

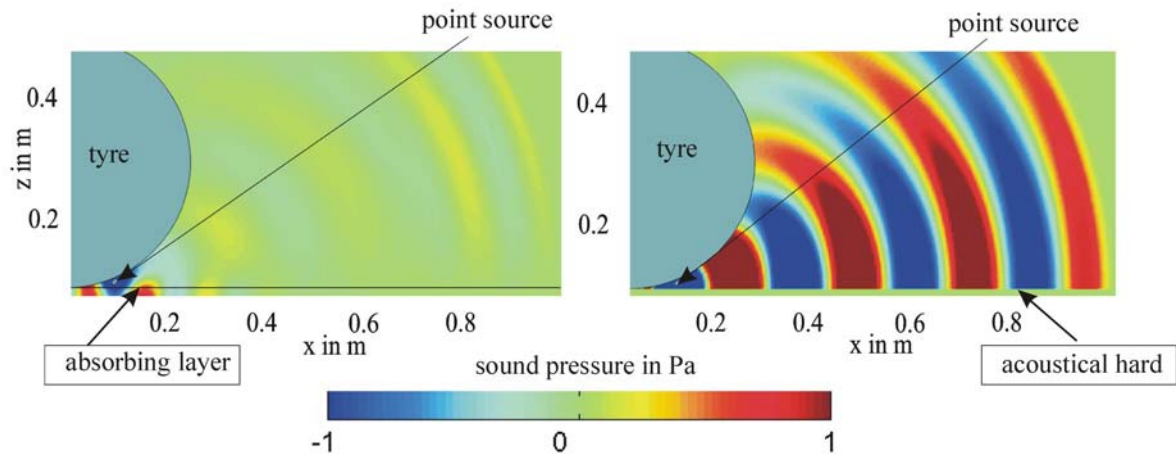


Figure 6. TLM-model. Sound field above an absorbing layer thickness 0.03m (left hand side) and above a reflecting layer (right hand side) calculated at a frequency of 1000 Hz.

Parameter of the absorber: flow resistivity $\Xi = 6000 \text{Ns/m}^4$, open porosity $\sigma = 22\%$, tortuosity $\tau = 5$.

Using the impedance Z_w , the reflection coefficient and the absorption of the surface of the absorbing layer can be calculated. In **Figure 5** the results of the analytical model are compared with the predictions of the TLM-model. The distance of nodes in the TLM-model depends on the wave length of the sound wave in the absorbing layer. To get less dispersion, the distance between the nodes is the twentieth part of the wave length in the absorber. In the lower frequency range the results agree rather good with the results of the analytical solution. For higher frequencies a deviation can be observed. The reason of this deviation hasn't been studied on a systematic way yet. But according to [4] it has to be mentioned that for each additional capacitance and inductance a parasitic inductance or parasitic capacitance is included, respectively.

In **Figure 6** the sound propagation above an absorbing road layer compared to the propagation above an acoustical hard reflecting layer is shown. The attenuation of SPL due to the usage of the open porous road surface in comparison to an acoustical hard surface is -20 dB at a frequency of 1000 kHz. The parameter and the thickness of the absorber are given in **Figure 6**. It is well known, that this strong attenuation occurs only at discrete frequencies due to destructive interference at the surface. For a local reacting absorber this interference arises if the thickness of the hard backed absorber layer is a quarter of the wave length of the sound wave within the absorbing layer. If the absorber is modeled as extended reacting this frequency shifts slightly.

Although at this stage of investigation the results of the TLM-model cannot be compared with that of the analytical model (the dimensions of the models are not in coincidence TLM 2-D, Analytical 3-D) and the accuracy of the TLM-model isn't known exactly, the results calculated with the TLM confirm the assumption of the extended reaction of the absorber. This can be observed in **Figure 6**, the sound wave within the absorber is reflected back into the sound field within air at a different location as the impinging wave.

4. CONCLUSION

The determined characteristic values of the porous road surface, the complex characteristic impedance and the complex wave number lead to the assumption that the road surface has to be modeled as extended reacting. For the examination of the extended reaction a TLM-model was set up. This model was extended to lossy media. In difference to other TLM-models of lossy media here the consideration of both characteristic values has been enabled.

For the validation of the TLM-model the absorption of the surface of a porous absorber for perpendicular incidence of a plane sound wave has been calculated. The results have been compared with that of an analytical model. In the lower frequency range the results agree rather good. Although for each frequency calculated the distance of the nodes within the mesh is adjusted to a twentieth of the wavelength within the absorber deviations can be observed at higher frequencies. The reason of this deviation hasn't been studied on a systematic way yet. But according to [4] it



has to be mentioned that for each additional capacitance and inductance a parasitic inductance or parasitic capacitance is included, respectively.

In spite of this deviations a model with a spherical wave front at nearly grazing sound incidence has been set up. Here the extended reaction of the absorber can be observed. Moreover an attenuation of -20 dB at the discrete frequency of 1000 Hz has been predicted. This attenuation occurs, when the wave length within the absorber is a quarter of the thickness of the hard backed absorber layer.

Because the dimensions of extended classical "Weyl von de Pol" solution and the TLM-model aren't in coincidence (TLM 2-D, Analytical 3-D) at this stage of examination the calculated results of the models cannot be compared. Moreover in future the accuracy of the TLM-model has to be studied systematically.

Using the TLM-model time transients for a discrete frequency can be considered. For the description of the alternation of location of a sound source (e.g. rolling tyre) the model has to be extended to broadband excitation. Here the frequency dependents of the characteristic values of the absorber and the possibility of modeling the group delay has to be examined.

ACKNOWLEDGEMENTS

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